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DYNAMIC PROGRAMMING

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SUMMARY

This is a brief description of decision processes in dynamic programming. ~~to appear in the SIAM Newsletter~~ early in 1956.

DYNAMIC PROGRAMMING

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It can be said that a field of human endeavor may be called a science to the degree that it is quantitative, and an art to the degree that it is qualitative. Although decision-making is essentially an art at the present time, and will remain so for some time to come, quantitative elements are beginning to enter to a greater and greater degree.

To determine the soundness of our judgment, we must inevitably compare predictions with results, an ofttimes expensive form of experimentation. A relatively inexpensive way is to construct a mathematical model of the underlying process, to compute the quantity which corresponds to the outcome of the original process, and, in this way, to compare theory and experiment.

Since we are usually interested in maximizing an output, or minimizing a cost, we encounter in this fashion a class of variational problems. Consider, for instance, a situation in which we must make a number of decisions at each stage of a process. Since each decision corresponds to a choice of a variable, we are confronted with a multidimensional problem in which the total number of variables is equal to the number of decisions to be made at each stage multiplied by the number of stages. Thus, for example, two decisions at each stage of a ten-stage process yield a twenty-dimensional variational problem.

Under certain simplifying assumptions where all the functions which arise may be taken to be linear, there are powerful methods, such as the simplex method of George Dantzig, for solving problems of this variety even in situations involving hundreds of variables. In addition there are iterative techniques, such as the "flooding technique" of Alexander Boldyreff, designed specifically for certain classes of processes.

If, however, the processes possess stochastic or nonlinear features, methods of the above variety are usually helpless, and we are left with formidable computational problems.

Nonetheless, quite often we are rescued by the fact that we are investigating a special class of multidimensional problems arising from multistage decision processes, and not completely general multidimensional problems. From the principle of wishful thinking, we deduce that there should be some means of making use of this fact. Consequently, in the above example there should be a method of formulating the mathematical problem so as to reduce it to a two-dimensional problem.

In order to do this, we must change our concept of a solution. In place of asking for a method which yields the entire sequence of twenty choices at one time, as the classical methods do, let us ask for a method which tells us the optimal decisions to make at each stage of the process in terms of the current situation. This is precisely the concept of a "policy" in the physical world.

The basic principle which enables us to formulate the functional equations which determine optimal policies is the following:

PRINCIPLE OF OPTIMALITY: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

A further advantage of the above approach, apart from the drastic reduction in dimensionality, is that it is ideally suited to processes involving random effects. Furthermore, it provides a new approach to the calculus of variations which enables us to resolve by numerical means, and occasionally analytic means, a variety of problems arising in applications which cannot easily be treated by other means.

Applications of the theory of dynamic programming have been made to finite-dimensional maximization problems, the calculus of variations, industrial production and allocation problems, scheduling theory, logistics, control processes, multistage games, and to a variety of military problems. A bibliography of papers up to 1954 may be found in

Bellman, R., "The Theory of Dynamic Programming," Bull. Amer. Math. Soc., Vol. 60, 1954, pp. 503-516.